Effects of Initial Waviness on the Strength and Design of Built-up Structures

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Compression strengths and optimum designs are investigated for lattice columns and truss-core sandwich panels whose principal load-carrying elements are initially wavy. Their strengths are decreased by waviness, more for the columns than the panels. For small amplitudes of initial waviness, optimum design occurs when the local is greater than the general buckling strength, as previous investigators have found. However, for larger amplitudes, and especially for the sandwiches, optimum designs occur when general is greater than local buckling strength. The major conclusion is that in neither case is the penalty great for using the conventional practice of equating the two modes.

Introduction and Summary

VAN der Neut established that when the bending stiffness of a built-up column is provided by thin plates, the initial waviness of those plates can result in a significantly reduced Euler strength for that column.[1] Based on these results, Thompson and Lewis investigated the minimum-weight design of that type of built-up column. In their investigation the ratio of Euler strength to local instability strength x is the chief design parameter.[2] They show that in the presence of moderate initial waviness, the column should be so proportioned that $x < 1$. It is also stated in Ref. 2 that "imperfection sensitivity is fundamentally more severe for $x > 1$ than it is for $x < 1$."

These investigations lead to several questions. For example, what are the effects of local initial waviness on other stability-designed structural configurations? Are the strengths of columns and other structural configurations, as proportioned by conventional optimization techniques, significantly less than those proportioned to account for initial waviness? Is "imperfection sensitivity...fundamentally more severe for $x > 1$..." for structures other than the particular type of column analyzed in Refs. 1 and 2?

To explore these questions, two types of structures are analyzed. The first is a lattice column whose longerons, but not whose axis, are initially wavy. It is established by an accurate analysis that the strength of that structure is even more sensitive to initial waviness than the columns in Refs. 1 and 2. The second is a uniaxially compressed truss-core sandwich panel that is long and simply supported along its unloaded edges. Its face plates are locally wavy, but its midplane is flat. By an approximate analysis, its strength is shown to be much less sensitive to local initial waviness than the column's. Moreover, that sensitivity is shown to be diminished when the panel is so proportioned that the ratio of Euler to local instability strength is greater than unity, in contrast to trends for the column examples.

The present two structures and the columns of Ref. 2 are also examined to determine the penalties for proportioning them according to the conventional $x = 1$ approach, rather than at the optimum value of x for a specified waviness. It is observed that the penalties are generally small for all three types of structures.

Conclusions are thus drawn that initial waviness does not especially invalidate the conventional approach for proportioning the structures for minimum weight; however, it does cause significant strength reduction. The extent of strength reduction and trends for optimum proportioning depend on the particular type of structure. Additional exploratory and detailed investigations are required to establish effects of initial imperfections on other types of stability-designed structures.

Lattice Column Analysis

The lattice column investigated is shown in Fig. 1, where A and I are the area and moment of inertia of a longeron, and $L \gg l$. It is assumed that the mode of failure is elastic instability in the plane of the lattice, that effects of transverse shear are negligible, and that the total weight of the column is proportional to the weight of the longerons.

Denote the overall buckling load as K_b, the load which produces local buckling of the longerons as K_l, and the Euler load as K_E. Then, for a material with Young's modulus of E,

$$K_E = (\pi^2 E A b^2 / 2L^2) \quad (1)$$

$$K_l = (2\pi^2 EI/l^2) \quad (2)$$

where it is assumed that the ends of the overall column are simply supported and that the batten members enforce nodes

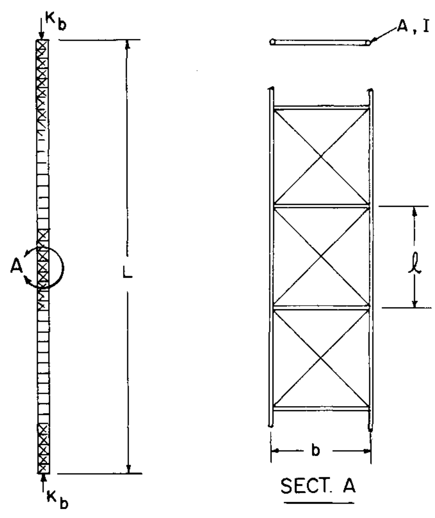

Fig. 1 Lattice-column structure.

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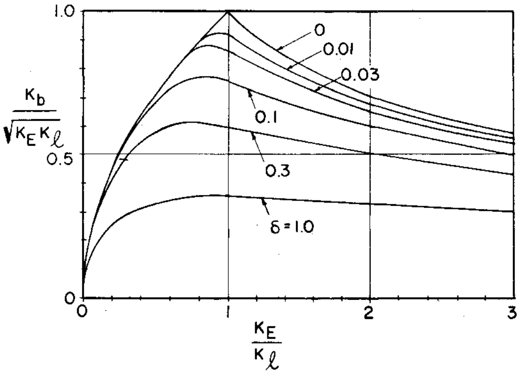

Fig. 2 Effect of local initial imperfections on optimum design of a lattice column.

and simple-support conditions for local buckling of the longerons.

Consistent with Ref. 2, the quotient K_E/K_l is used as a measure of bay length l, and constant proportions and weight are assumed. For the lattice column,

$$(K_E/K_l) = (A\beta^2/4IL^2)l^4 \tag{3}$$

where

$$\beta = b/l$$

Note that the assumed condition for the present analysis is that l is the only parameter that will be varied in determining the maximum value of K_b, and that β, L, I, and A are held constant.

The overall buckling load would be the smaller of K_E and K_l if no initial imperfection existed. When both local and overall imperfections are present, the analysis of their effects is quite complex, as illustrated in Ref. 3 for a triangular lattice column. However, when either type of imperfection is present alone, the analysis is straightforward. Only local imperfections of the lattice column are considered here, however, in consonance with the approach in Ref. 2.

For this case, the overall buckling load can be found by the Shanley column formula

$$K_b = \frac{\pi^2 E_{\tan} A b^2}{2L^2} \tag{4}$$

where $E_{\tan}$ is the effective tangent modulus of longerons with initial imperfections. This quantity can be determined by assuming sinusoidal initial imperfection of half-wavelength l in the longerons, and then finding the resulting relationship between axial load and shortening. The result is

$$E_{\tan} = E\bigg/\left(1 + \frac{\delta^2}{2}\frac{1}{(1-P^*)^3}\right) \tag{5}$$

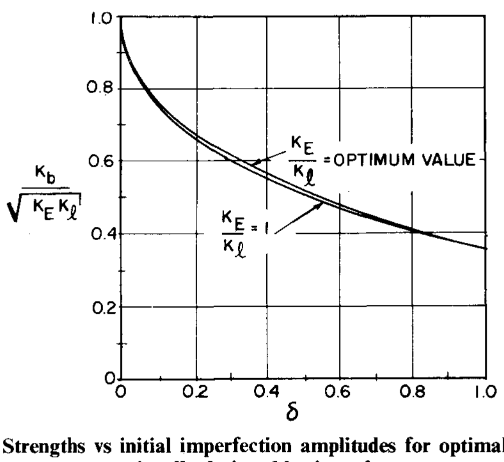

Fig. 3 Strengths vs initial imperfection amplitudes for optimally and conventionally designed lattice columns.

where P^* is the ratio of the longeron load to its local buckling load, and δ is the initial-imperfection amplitude divided by the longeron bending radius of gyration $(I/A)^{1/2}$. Thus,

$$P^* = (K_b/K_l) \tag{6}$$

The objective is to select l to maximize K_b for the specified overall weight, proportions, and initial imperfection. For ease in computation, the quantity P^* can be viewed as the parameter on which the others depend. For example, the bay-length measure is

$$K_E/K_l = (K_b/K_l)(K_E/K_b) = P^*(E/E_{\tan}) \tag{7}$$

From Eq. 5, $E/E_{\tan}$ is seen to be a function of P^*. The quantity

$$\frac{K_b}{(K_E K_l)^{1/2}} = \frac{K_b L}{\pi^2 E(AI\beta^2)^{1/2}} \tag{8}$$

is nondimensional and the product $K_E K_l$ is independent of the bay length. The nondimensional maximum load can be expressed in terms of P^*, using Eqs. 6 and 7 as follows

$$\frac{K_b}{(K_E K_l)^{1/2}} = P^*\left(\frac{K_l}{K_E}\right)^{1/2} = \left(P^*\frac{E_{\tan}}{E}\right)^{1/2} \tag{9}$$

The optimization is therefore performed by plotting $K_b/(K_E K_l)^{1/2}$ vs. K_E/K_l for specified values of δ, and selecting the maxima of these curves.

The resulting curves are shown in Fig. 2, as calculated by using Eqs. (5, 7 and 9). The character of these curves is different from those in Fig. 7, Ref. 2 (for a column whose bending stiffness is provided by panels rather than longerons), in at least two ways. First, the present curves always have maxima, even for large initial imperfections, while the maxima cease to exist for panel columns when the panel imperfections are large. Second, the strength reduction is larger in the present case for equal amplitudes of initial imperfections. For example, at $\delta = 0.3$, the maximum buckling load is 61% of the maximum possible for no imperfections. For a square cross section, Van der Neut's imperfection parameter α relates to the present δ by the formula

$$\alpha = [\delta/[2(3)^{1/2}]$$

since α is nondimensionalized with respect to the thickness rather than the bending radius of gyration. Thus, when $\delta = 0.3$, $\alpha = 0.0866$ and the maximum column load[2] is 83% of the perfect-column result.

To demonstrate the effectiveness of designing imperfect lattice columns at their optimum value of K_E/K_l for a prescribed value of δ, two curves are plotted in Fig. 3 for $K_b/(K_E K_l)^{1/2}$ vs δ: a curve of the optimum values of K_E/K_l, and a curve for $K_E/K_l = 1$. Figure 3 shows that the gain is very small when the column is designed at true optimum rather than at the conventional $K_E/K_l = 1$ optimum.

Truss-Core Sandwich Panel Analysis

The truss-core sandwich panel analyzed here is shown in Fig. 4. It is assumed to be uniaxially compressed, of width b, and simply supported along its unloaded edges. The facings are of thickness t and are assumed to be simply supported by the core along parallel lines separated by distance d, where $d \ll b$. The sandwich thickness h is assumed to be proportional to d.

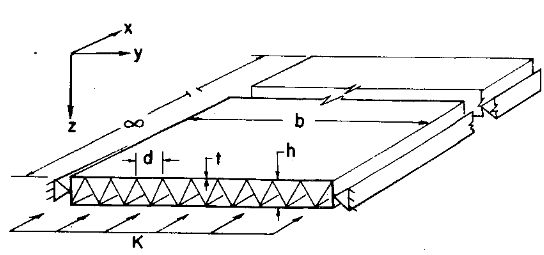

Fig. 4 Truss-core sandwich panel.

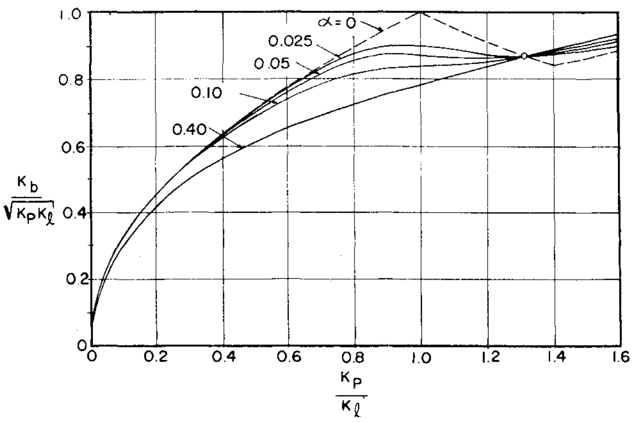

Fig. 5 Effect of local initial imperfections on design of truss-core sandwich panels.

The objective of this analysis is similar to that of the foregoing: to maximize the overall elastic buckling load of the sandwich panel for a specified overall panel weight, panel width, detailed proportion, and local initial imperfection. Accordingly, t, d/h, and b are held constant, and d is the only parameter that is varied. Note that the plane of the sandwich is assumed to be initially flat, so that only local imperfections of the facing sheets enter this exploratory analysis.

Both panel- and local-instability loads, K_P and K_l respectively, are given in Ref. 4 by the formulas

$$K_P = \frac{2\pi^2}{b}\left[(D_x D_y)^{1/2} + D_3\right] \tag{10}$$

and

$$K_l = \frac{8\pi^2 Ebt^3}{12(1-\nu^2)d^2} \tag{11}$$

D_x, D_y, and D_3 in Eq. (10) are the flexural and twisting stiffnesses of the overall sandwich and are defined by the following constitutive relationships among the moments and curvatures

$$M_x = D_x\left(\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2}\right) \tag{12}$$

$$M_y = D_y\left(\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2}\right) \tag{13}$$

$$M_{xy} = 2D_{xy}\frac{\partial^2 w}{\partial x\,\partial y} \tag{14}$$

and

$$D_3 = \nu_x D_y + D_{xy} \tag{15}$$

When the facings have no initial waviness and no plasticity, and the presence of the truss core is neglected, then

$$D_x = D_y = D_3 = D = \left[Eth^2/2(1-\nu^2)\right] \tag{16}$$

where ν is Poisson's ratio for the facing material. Then K_p becomes

$$K_P = \frac{2\pi^2 Eth^2}{(1-\nu^2)b} \tag{17}$$

Consistent with the approach in the foregoing column analysis, the quotient K_P/K_l is used as a measure of the core spacing d

$$\frac{K_P}{K_l} = 3\frac{d^4}{b^2t^2}\left(\frac{h}{d}\right)^2 \tag{18}$$

Note that d is the only variable in this equation since h/d, b, and t are held constant.

Initial waviness is assumed to be present in the facings, and its mode is assumed to be the same as that for the minimum local buckling load. The effect of that waviness on overall panel buckling is accounted for in an approximate manner by reduction factors η_x, η_y, and η_3 on the stiffnesses in Eq. (10). The buckling load K_b is then

$$K_b = \frac{2\pi^2}{b}\left\{\left[(\eta_x D_x)(\eta_y D_y)\right]^{1/2} + \eta_3 D_3\right\}$$
$$= \frac{2\pi^2 D}{b}\left[(\eta_x \eta_y)^{1/2} + \eta_3\right] \tag{19}$$

The reduction factor η_x is the same as the η factor given by Van der Neut[1] by which the in-plane stiffness of a panel in the loaded direction is reduced because of initial waviness. It does not appear that analyses exist to determine η_y, which is the reduced in-plane stiffness of a wavy panel in the transverse direction to the load. However, it is clear that the stiffness in the unloaded direction will be greater than that in the loaded direction. It is therefore conservatively approximated that they are equal; then,

$$\eta_x = \eta \tag{20}$$

and

$$\eta_y = \eta \tag{21}$$

Regarding η_3 in Eq. (19), it is approximated that

$$\eta_3 \simeq 1 \tag{22}$$

because D_3 is largely D_{xy}, and D_{xy} would not be greatly reduced by the present initial waviness mode, which is quite different from the natural mode for local shear buckling of facings. Accordingly, Eq. (19) becomes

$$K_b = (1+\eta/2)K_P \tag{23}$$

The load at which the sandwich buckles is calculated in the nondimensional form $K_b/(K_P K_l)^{1/2}$, where the product $K_P K_l$ is seen to be independent of the variable d since the ratio d/h is assumed to be fixed.

The optimization is performed as before by plotting $K_b/(K_P K_l)^{1/2}$ vs K_P/K_l for specified values of an initial waviness parameter and by noting the maxima. Van der Neut's initial waviness parameter α[1] is used here, where

$$\alpha = (a/t) \tag{24}$$

and a is the waviness amplitude. The computations are made by reading values of η from Ref. 1 for an assumed value of K_b/K_l. Then K_P/K_l is determined from the formula

$$K_P/K_l = (K_P K_b)(K_b/K_l) = 2(K_b/K_l)/(1+\nu) \tag{25}$$

Finally,

$$\frac{K_b}{(K_P K_l)^{1/2}} = \frac{K_b}{K_l}\left(\frac{K_l}{K_P}\right)^{1/2} = \left(\frac{1+\eta}{2}\frac{K_b}{K_l}\right)^{1/2} \tag{26}$$

is calculated.

The results of such calculations are presented in Fig. 5 as a graph of $K_b/(K_P K_l)^{1/2}$ vs K_P/K_l for various values of α. The character of these curves is seen to be substantially different from both those of the foregoing lattice column and those of the column analyzed in Ref. 2. First, the reduction in maximum strength of a sandwich panel is less for a given value of α than it is for either type of column. Second, for a sandwich panel the effects of initial waviness are less severe when K_P/K_l (compare with K_E/K_l in the column analyses) is greater than unity than when it is less than unity. This trend contrasts with that shown for columns, both here and in Ref. 2. In fact, Fig. 5 indicates that the strengths for sandwich panels (especially for large values of α) might be maximized in the post-local-buckling region, if the secondary strengths of the structure will tolerate post-buckled operations. Such tolerance could depend on the primary operating stresses; therefore, that tolerance would also depend upon the loading index for which the sandwich is designed.

Conclusions

Effects of localized initial waviness are analyzed for built-up columns and sandwich panels whose designs are governed by elastic stability considerations. It is demonstrated that the effects can be substantially different for the two different types of structures. For the lattice columns, large strength reductions will be caused, while for the sandwich panels, the same amounts of local waviness cause strength reductions that are only moderate. The strength reductions for the plate-stiffened columns of Ref. 2 are intermediate relative to the presently analyzed structures.

It is shown that the efficiency of lattice columns is not significantly compromised by using conventional design procedure (equating local and Euler instability loads) rather than optimum design procedures which account for initial waviness. The same trend holds true for sandwich panels, except when the initial waviness is of large amplitude. It is similarly obvious from the results of Ref. 2 that the efficiency of plate-stiffened columns is not significantly compromised by using conventional design practice.

It is therefore concluded that the effects of initial waviness on the buckling strength of built-up structures depend on the particular type of structure. Each type must be investigated separately, and many other types are still to be investigated. However, from the common trends observed here, no significant increases in efficiency are to be gained through optimizing structural proportions for a specific imperfection amplitude, regardless of the type of structure involved.

It is also observed from these studies that different types of structural elements have differing reductions in their stiffnesses for equal amplitudes of initial waviness. This accounts for the varying strength reductions among the structures analyzed here and in Ref. 2.

There still remains a wide variety of structural elements whose reductions in stiffness should be determined for various conditions of initial waviness and under various loadings. Such reductions can be important not only to the static buckling strengths but also to the vibrational and aeroelastic characteristics of the structural assemblies in which those elements are used.

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On a Numerical Sufficiency Test for Monotonic Convergence of Finite Element Models

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Finite element analyses characterized by monotonic convergence include the discipline for meaningful measurements of convergence rate and consequently economical extrapolation. Few proposers of element models guarantee monotonic convergence for their elements. Thus, a need exists for an automatic test to classify available element models. This paper describes such a test—a test that can be performed using a digital computer to guarantee that a particular element model imbues monotonicity. It describes the test and its basis. It examines seven element models for a rectangular membrane to illustrate the value of the tests. Besides confirming results already known, the application yields new data. It "proves" monotonicity for two improved models, defines the range of element proportions for which another element can be guaranteed to exhibit monotonicity, and suggests that another element is deficient. In the special case of absolutely convergent membrane displacement models, proof of monotonicity is a necessary and sufficient condition to insure that upper bound estimates of strain energy are developed. Accordingly, the test furnishes a proof of bound solutions independently of requirements on displacement continuity the element basis may or may not satisfy.

Introduction

IT is important that a finite element analysis model exhibit monotonic convergence. By definition this guarantees an equal or reduced error norm for successive analyses with finer meshes. A desirable consequence of monotonic convergence is the potential for drastically improving analysis efficiency by using extrapolation techniques. In addition, monotonicity is necessary if the rate of change of the norm is used as a means of deducting the magnitude of discretization error. A need exists for an experimental test basis of monotonic convergence. Such a test would facilitate evaluating convergence characteristics of existing and proposed element models.

Guarantees of monotonic convergence for most of the elements in the literature are not currently available, perhaps because of the difficulty of proving such convergence analytically. Similarly, computer code technical documentation describing element models rarely testifies to monotonic convergence. Experience indicates that in at least two very widely used codes, elements are available which disrupt monotonic convergence of the set.

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